Worksheet Twelve **MATH 104** Name:________________

WORK ON PROBLEMS IN GROUP OF 2-4. YOUR INSTRUCTOR WILL MARK YOUR GROUP WORK IN CLASS. TURN IN YOUR OWN WORK FOR QUESTIONS MARKED AS "INDIVIDUAL WORK" INDIVIDUALLY; UPLOAD TO CANVAS OR SUBMIT IN CLASS ON THE DUE DATE.

9.6: Solving Systems Using Gaussian Elimination

- Matrix of coefficients: Write the system so that each equation is in its own line. Order the equation with variable on one side and constants on the right side. Place x variable of each equation in one column and y variable of each equation in one column and z variable of each equation in one column leaving empty spots if a variable is missing in an equation. Form a matrix out of the coefficients only, writing zeros for the empty spots.

- Vector of constants: Write the system as mentioned above. The column of all constant terms form an $n \times 1$ matrix that we call matrix of constants.

- Augmented matrix: The matrix resulting from joining the matrix of coefficients and the matrix of constants is called augmented matrix. In the augmented matrix, we draw a vertical line between the two matrices. The line does not exist when doing matrix operations with your calculator. We use augmented matrices instead of systems to perform elimination methods and solve the systems.

- Gaussian Elimination:

  Gaussian elimination is using row operations to eliminate at least one variable in the second row; at least two variables in the third row and so on. n an elimination step, the rows above the row in which the elimination is happening must be used.

  At the end we like to have an **upper triangular form** for the matrix of coefficients. This is also called row echelon form of the matrix.

$$\left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 1 & a_{23} & b_2 \\ 0 & 0 & 1 & b_3 \end{array}\right] \text{(a unique solution)}$$

$$\text{or } \left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 1 & a_{23} & b_2 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{(infinitely many solutions) or } \left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 1 & a_{23} & b_2 \\ 0 & 0 & 0 & b_3 \end{array}\right] \text{(inconsistent if } b_3 \neq 0)$$

$$\text{or } \left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 0 & 1 & b_2 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{(infinitely many solutions) or } \left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 0 & 1 & b_2 \\ 0 & 0 & 0 & b_3 \end{array}\right] \text{(inconsistent if } b_3 \neq 0)$$

$$\text{or } \left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{(infinitely many solutions) or } \left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 0 & 0 & b_2 \\ 0 & 0 & 0 & b_3 \end{array}\right] \left(\begin{array}{c} \text{inconsistent if} \\ b_2 \neq 0 \text{ or } b_3 \neq 0 \end{array}\right)$$

- Reminder: Row Operations are

  (1) Exchange rows.

  (2) Add a multiple of row to another.

  (3) Multiply a row by a number.

- Solving a system with a unique solution using the row echelon form:

  Rewrite in the system form:

$$\begin{array}{rrrl} x & +a_{12}y & +a_{13}z & = b_1 \\ & y & +a_{23}z & = b_2 \\ & & z & = b_3 \end{array}$$

  Use the last row to solve for z, then replace z in the second row to solve for y. Last, replace both y and z in the first row to solve for x.

  The first non zero coefficient in each row should turn into a one, using a row operation, which is called a **leading one**.

- Note: If every variable has a leading one in their column, then the system has a unique solution. If the system has infinitely many solutions, then at least one column does not contain a leading one. Assign a parameter, t, to the variable associated with that column. Then solve for the rest of the variables. (This suggestion is a bit different from your book's method but very effective when you learn about larger systems.)

- Another note: Row Echelon form of a matrix is not unique. For a system of 3 equation and 3 variables, its form will be unique up to one of the forms in the previous page but note that a_{ij}'s and b_i's may vary.

Related Videos:

1. Example 1: https://mediahub.ku.edu/media/MATH+-+Gaussian+Elimination.m4v/1_806uqq36

2. Example 2: https://mediahub.ku.edu/media/t/1_8wx2imvq

1. Find a row echelon form of the following matrix. Show all your work. Explain every elementary row operation you used. $\left[\begin{array}{ccc|c} 0 & 1 & 5 & 75 \\ 0 & 0 & 4 & 52 \\ 3 & 15 & -24 & 0 \end{array}\right]$

2. Find a row echelon form of the following matrix. Show all your work. Explain every elementary row operation you used. $\left[\begin{array}{ccc|c} 8 & 1 & 5 & 75 \\ 0 & 1 & 4 & 52 \\ 3 & 15 & -24 & 0 \end{array}\right]$

3. What is a row equivalent system of equations to the following augmented matrix? What is the solution to that system?

$$\left[\begin{array}{ccc|c} 1 & 3 & -7 & 0 \\ 0 & 1 & 6 & -4 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

4. What is a row equivalent system of equations to the following augmented matrix? What is the solution to that system?

$$\left[\begin{array}{ccc|c} 1 & 3 & -7 & 0 \\ 0 & 1 & 6 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

5. What is a row equivalent system of equations to the following augmented matrix? What is the solution to that system?

$$\left[\begin{array}{ccc|c} 1 & 3 & -7 & 0 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

6. Consider the system

$$\begin{cases} x & +3y & +2z & = 8 \\ -2x & -4y & -z & = -4 \\ x & -y & +z & = -6 \end{cases}$$

(a) Find the augmented matrix associated with this system.

(b) Find a row echelon form of the augmented matrix. Show all steps. Explain what elementary row operations you used in each step.

(c) Use the row echelon form to write the row equivalent system of equations.

(d) Solve the system or find that it is inconsistent.

7. Consider the system

$$\begin{cases} x & +3y & +2z & = 8 \\ 2x & -4y & -z & = -24 \\ x & -7y & -3z & = -33 \end{cases}$$

(a) Find the augmented matrix associated with this system.

(b) Find a row echelon form of the augmented matrix. Show all steps. Explain what elementary row operations you used in each step.

(c) Use the row echelon form to write the row equivalent system of equations.

(d) Solve the system or find that it is inconsistent.

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INDIVIDUAL WORK

UPLOAD TO CANVAS OR SUBMIT IN CLASS BEFORE DUE DATE. DISCUSSING THESE QUESTIONS IN YOUR GROUP IS ENCOURAGED BUT MAKE SURE YOU ARE TURNING IN YOUR OWN WORK.

8. Consider the system

$$\begin{cases} x & +3y & +2z & = 8 \\ 2x & -4y & -z & = -24 \\ x & -7y & -3z & = -32 \end{cases}$$

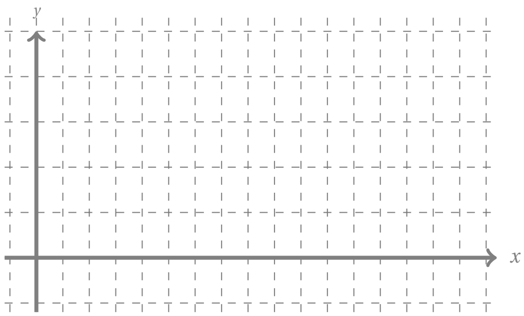
(A) *(1 point) Find the augmented matrix associated with this system.*

(B) *(2 points) Find a row echelon form of the augmented matrix. Show all steps. Explain what elementary row operations you used in each step.*

(C) *(1 point) Use the row echelon form to write the row equivalent system of equations.*

(D) *(1 point) Solve the system or find that it is inconsistent.*

9. *(1.5 points) Shade the solution to* $\begin{cases} 0 \le x \le y^2 \\ 0 \le y \le 4 \end{cases}$.



10. *(1.5 points) Shade the solution to* $\begin{cases} 0 \le x \le 16 \\ \sqrt{x} \le y \le 4 \end{cases}$.